Upper Air Sounding Analysis by Use of an Electronic Computer

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(Manuscript received 16 July 1965, in revised form 10 February 1966)

ABSTRACT

The upper air sounding analysis technique currently employed by the Severe Local Storms Forecasting Unit of the U. S. Weather Bureau at Kansas City has been programmed for computation by an electronic computer. Equations used and program routines are discussed.

1. Introduction

The Severe Local Storms Forecasting Unit (SELS) of the Weather Bureau at Kansas City analyzes upper air soundings as part of its routine forecasting procedure. In the past, the forecaster would select an area of interest, as determined from surface and upper air charts, then plot and analyze soundings from this area. Hand plotting and analyzing by this method (U. S. Weather Bureau, 1956) is somewhat subjective and time consuming in that it requires equal-area averaging and judging proportional distances on pseudo-adiabatic charts.

Soundings are now analyzed by use of an electronic computer, with the advantage that more soundings can be made available at an earlier time. The method is objective and about four man-hours of plotting and analysis are saved each day.

The purpose of the radiosonde analysis is to give the forecaster a quick look at 1), the areas and magnitude of unstable air, and 2), the areas that may become unstable with daytime heating.

This paper describes the equation and program routines used in the machine method. The analysis technique is basically a parcel method. The parcel lifted is one which possesses moisture and potential temperature values equal to the average values of these parameters found in the lower stratum of the atmosphere. This lower stratum or surface layer is defined as the lowest 100 mb of the sounding.

Input data for the program are temperatures and dew points for mandatory levels and pressures, and temperatures and dew points for significant levels from the surface to 400 mb.

Output data from the program are: lifted parcel temperature at 500 mb; pressure at the level of free convection, if one exists; lifted index, defined as the 500-mb temperature minus the lifted parcel temperature at 500 mb; average mixing ratio and average potential temperature of the surface layer; and the forecast maximum surface temperature. Also included in the output are parameters to be discussed later in the text, namely, overrunning pressure, potential hail size, potential gust velocity and warm air capping in the mid-levels.

2. Program routines

The first routine decodes the temperatures and dew points. Negative temperatures have 50 subtracted from their magnitude and a negative sign prefixed.

Next, the data are arrayed in order of descending pressure. Mandatory pressures are not punched in the input data, and it is necessary to define these pressures (1000 mb, 850 mb, etc.) and match them with their temperatures and dew points. Certain pressure levels need to be identified for future use (400 mb, 500 mb and the pressure at the top of the surface layer). As the levels are arrayed in order of descending pressure, these desired levels are assigned a control index for identification.

After the pressures are arrayed, the average mixing ratio and average potential temperature of the surface layer are computed. Rarely will a mandatory or significant level fall at the top of the surface layer. Therefore, a temperature and dew point for this level are determined by interpolation. Fig. 1 is a schematic of the routine. The temperature at the top of the surface layer $T(N)$ (°C) is given by the expression

$$T(N)=T(P2)-\frac{\Delta T}{\Delta P_1}\Delta P_2,$$

$$=T(P2)-\frac{T(P2)-T(P3)}{P2-P3}\times(P2-PN). \quad (1)$$

Similarly, the dew point $D(N)$ (°C) is

$$D(N)=D(P2)-\frac{D(P2)-D(P3)}{P2-P3}\times(P2-PN), \quad (2)$$

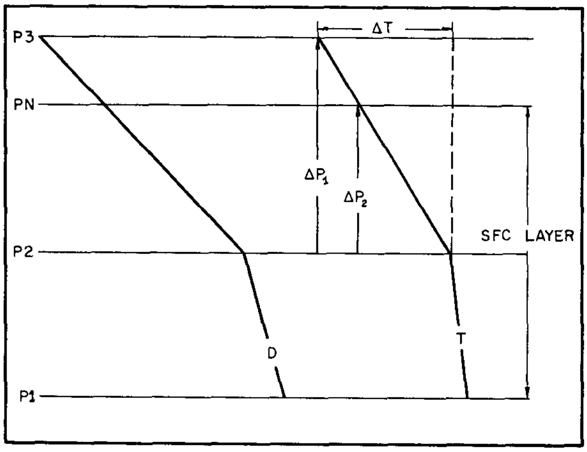

FIG. 1. Interpolation routing for obtaining temperature and dew point at top of surface layer.

where the terms in (1) and (2) are illustrated in Fig. 1. It is assumed that pressure decreases linearly instead of logarithmically in all interpolation routines. Comparisons of the two methods have been made and the differences are not found to be significant over the short intervals used for this particular application. It is further assumed that the pseudo-adiabatic curve is linear for short intervals. Interpolations between this curve and pressure are very good since the differences introduced by the linear process are opposite in sign and nearly equal in magnitude. Mixing ratios (w) are computed for all levels in the surface layer by the equation

$$w=0.622\left(\frac{e}{P-e}\right), \tag{3}$$

where e is the vapor pressure. Vapor pressure is computed by Tetan's equation described by Berry *et al.* (1945), i.e.,

$$e=6.11(10)^{7.5D/(237.3+D)}, \tag{4}$$

where D is the dew point in °C and e is in millibars.

Poisson's equation is used to compute the potential temperature of each level in the surface layer. The average potential temperature ($\bar{\theta}$) is found in a different manner on the 1200 GMT soundings (morning) than on the 0000 GMT soundings (evening). In the evening, no further heating is expected from insolation; therefore the existing potential temperatures are averaged. On the 1200 GMT soundings considerable surface heating may take place during the day and a maximum surface temperature is forecast. The average potential temperature of the surface layer is considered to be the one corresponding to the maximum forecast surface temperature. An empirical technique is used to forecast the maximum temperature for the day. Two degrees (C) are added to the temperature at the top of the surface layer. Dry adiabatic compression to the surface pressure yields the forecast maximum temperature.

After the averages for mixing ratio and potential temperature are determined for the surface layer, the program computes temperatures for a lifted parcel (*TLP*) based on these averages. To accomplish this, temperatures are computed at each pressure level corresponding to wet bulb potential temperatures of 10C, 20C, and 30C, using a third order polynomial for each curve. A wet bulb potential temperature is computed from the average mixing ratio and average potential temperature of the surface layer. The ratio established by this average wet bulb potential temperature and two of the three polynomial temperatures at 1000 mb is used to interpolate temperatures of the lifted parcel for the remaining pressure levels. Fig. 2 is a schematic of this procedure. The wet bulb potential temperature of the surface layer is computed by an empirical equation used by the Weather Records Processing Center, formerly at Kansas City, i.e.,

$$\theta_w=T-ab, \tag{5}$$

where

T = dry bulb temperature,
$a=0.035n-0.0072n(n-1)$,
$n=(T-D)/5.5555$,
D = dew point temperature (°C),
$b=T+D-(P/30.4775)+95.5555$,
P = pressure (mb).

The dew point at 1000 mb is computed by first solving the mixing ratio equation for vapor pressure and then solving Tetan's equation for the dew point temperature.

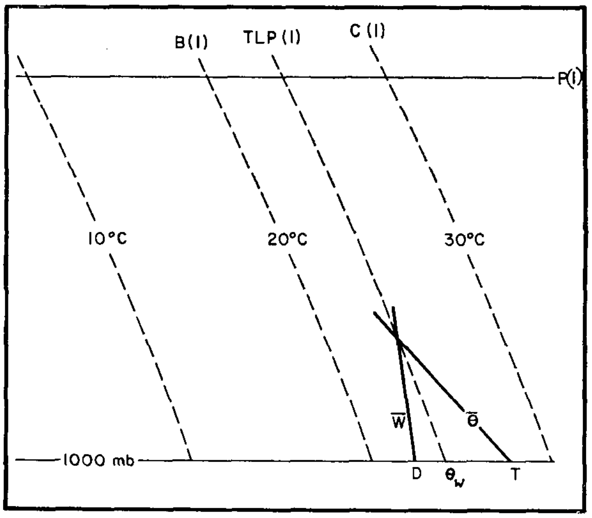

FIG. 2. Schematic of interpolation routine for determining temperatures of lifted parcel (*TLP*).

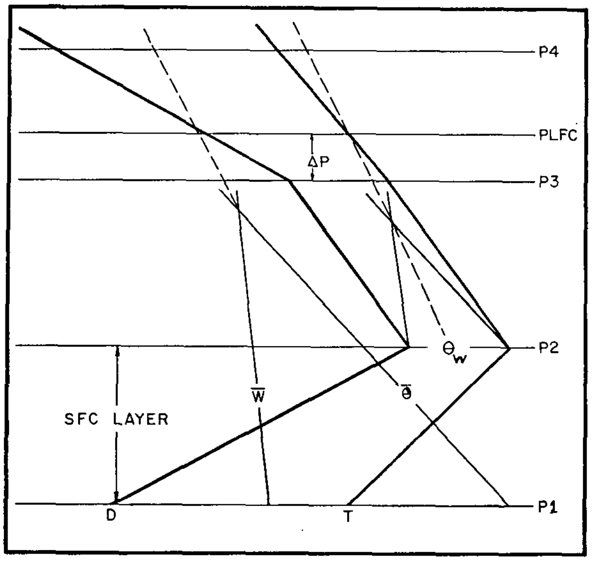

FIG. 3. Schematic for "overrunning" check, and interpolation routine for determining pressure at level of free convection (*PLFC*).

The temperature of the lifted parcel at any level (I) is given by the expression

$$TLP(I)=B(I)+\left(\frac{\theta_w-20}{10}\right)[C(I)-B(I)], \qquad (6)$$

where temperatures $B(I)$ and $C(I)$ are defined in Fig. 2, if the wet bulb temperature is greater than 20C. Should it be less than 20C, the interpolation would be between the polynomial curves for 10C and 20C.

In some cases the surface layer may be quite cold and dry while just above there is a layer of warm, moist air (Fig. 3). Lifting a parcel from the surface in this case would indicate a very stable sounding, while lifting from the top of the inversion layer might result in an unstable sounding. A check for this overrunning is performed next. Wet bulb temperatures are computed at each level above the surface layer and compared with the temperature of the lifted parcel. The level where the wet bulb temperature exceeds the temperature of the lifted parcel by the greatest amount, if one exists, is selected, and new temperatures of the lifted parcel are computed based on the higher wet bulb temperature. One should note that in the overrunning case temperatures of the lifted parcel and, hence, other derived values such as lifted index, hail size, etc., are now based on a single level aloft and not a 100-mb layer.

A search for the pressure at the level of free convection (*PLFC*) is made next. The dry bulb temperature is compared with the temperature of the lifted parcel at each level. The *PLFC* will generally come between pressure levels; however, should it fall on a data level, this level is selected and the search is over. Normally, the *PLFC* is found by interpolation between the first level where the *TLP* is warmer than *T* and the next lower level. In Fig. 3 the expression for *PLFC* is

$$PLFC=P(3)-\Delta P$$

$$=\frac{[P(3)-P(4)][TLP(3)-T(3)]}{[T(3)-T(4)]-[TLP(3)-TLP(4)]}+P(3). \qquad (7)$$

Similarly, the temperature at the pressure of free convection is

$$TLFC=T(3)-\frac{[T(3)-T(4)][P(3)-PLFC]}{P(3)-P(4)}. \qquad (8)$$

Potential hail stone size is computed next. The diameter of a hail stone is related to its terminal velocity. An object, when dropped through the atmosphere, will accelerate until the aerodynamic drag force is just equal to the weight of the object. This velocity is its terminal velocity. The diameter of a hail stone is related to its terminal velocity by the expression

$$w=\left(\frac{4\rho' dg}{3\rho C_D}\right)^{\frac{1}{2}}, \qquad (9)$$

where ρ' is the density of the hail stone, d is hail stone diameter, g is acceleration of gravity, C_D is drag coefficient and ρ is the density of air. A value of 0.9 gm cm^{-3} was used for the density of the hail stone, 0.6 for the drag coefficient, and the density of the air is computed from the pressure and temperature at the hail formation region. Vertical motion at the hail formation region derived from buoyancy is given by

$$w_H=\left(\frac{g}{T}\Delta T_H H\right)^{\frac{1}{2}}, \qquad (10)$$

where ΔT_H is the temperature difference between the parcel curve and the environment at the hail formation region (−10C on the lifted parcel curve), and H is the thickness between the level of free convection and pressure at the hail formation region. Eqs. (9) and (10) are combined and solved for hail stone diameter (d). Foster and Bates (1956) give a fuller discussion of these equations.

The maximum surface wind gust, after Foster (1958), is computed by assuming the downdraft parcel will follow a moist adiabat which represents the average between the lifted parcel curve (*TLP*) and the average wet bulb temperature ($\bar{\theta}_w$) for the layer 700–500 mb (Fig. 4). The negative buoyancy from the level of free sink (point at which the descending parcel becomes colder than the environment) to the surface is used to compute the surface gust. Eq. (10) is used where H is the thickness between the surface and the level of free sink and ΔT_H is the difference between the temperature of the descended parcel at the surface and the surface temperature corresponding to $\bar{\theta}$. The level of free

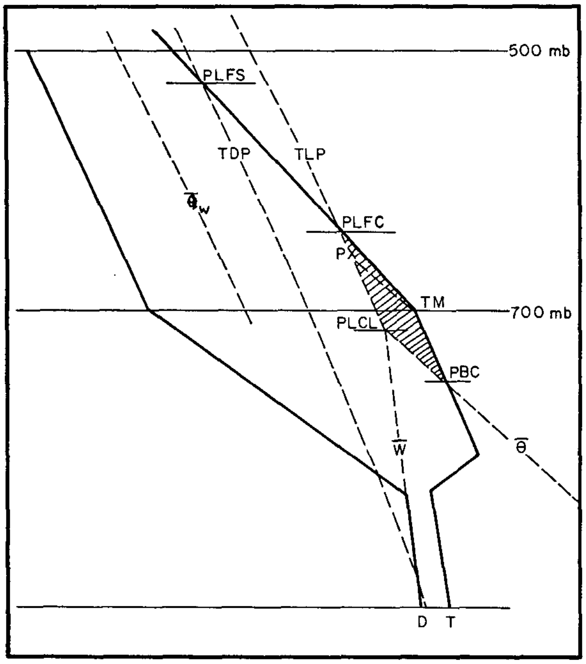

FIG. 4. Schematic for computing maximum surface gusts and amount of "capping" in mid-levels.

sink is determined by interpolation in the same manner as the level of free convection.

Finally, the amount of "capping" in mid-levels is computed (hatched area, Fig. 4). The cap is dimensioned by the pressure difference between the base of the cap (point at which the lifted parcel becomes colder than the environment) and the level of free convection and by the greatest temperature difference between the temperature of the lifted parcel and the environment. The pressure at the base of the cap (*PBC*) is found by interpolating between the first level where the lifted parcel is colder than the environment (700 mb in Fig. 4) and the next lower level. The interpolation technique is the same one employed for the pressure at the level of free convection and the level of free sink. The width of the cap is determined by selecting the greatest difference in comparing temperatures of the lifted parcel with environmental temperatures at each level within the capping layer. In order to compute the amount of lift necessary to eliminate the cap, it is necessary to compute the temperature (*TLCL*) and pressure (*PLCL*) at the lifted condensation level. As an unsaturated parcel ascends, its dry bulb temperature and dew point approach at a rate equal to the difference between the dry adiabatic lapse rate and the dew point lapse rate. The temperature (*T*) and dew point (*D*) at 1000 mb corresponding to the average mixing ratio and average potential temperature are used.

We may write

$$T_{1000}+\frac{dT}{dZ}\Delta Z=D_{1000}+\frac{dD}{dZ}\Delta Z,$$

where ΔZ is the height of the *LCL* above the 1000-mb surface. Rewriting,

$$\Delta Z=\frac{D_{1000}-T_{1000}}{\dfrac{dT}{dZ}-\dfrac{dD}{dZ}}.$$

Then

$$TLCL=T_{1000}+\frac{dT}{dZ}\Delta Z,$$

$$=T_{1000}+\left[\frac{\dfrac{dT}{dZ}(D-T)_{1000}}{\dfrac{dT}{dZ}-\dfrac{dD}{dZ}}\right]. \qquad (11)$$

The pressure at the lifted condensation level is found from Poisson's equation

$$\bar{\theta}=TLCL\left(\frac{1000}{PLCL}\right)^{R/c_p},$$

or

$$PLCL=1000\left(\frac{TLCL}{\bar{\theta}}\right)^{c_p/R}. \qquad (12)$$

A parcel with the environmental temperature (*TM*) at the greatest width of the cap is lifted dry adiabatically until it cools to the temperature of the lifted parcel (*TLP*) at pressure (*PX*). The difference in pressure (*PX*) and the pressure at (*TM*) is the amount of lift necessary to eliminate the cap. The form of the dry adiabatic lapse rate used comes from the first law of thermodynamics for constant entropy, i.e.,

$$dh=0=c_p dT-\alpha dp,$$

from which, with the gas equation,

$$\frac{dT}{dp}=\frac{RT}{c_pP}=K\frac{T}{P}=0.288\frac{T}{P},$$

where T is in °K. The interpolation for (*PX*) assumes the pseudo-adiabatic lapse rate is linear from the lifted condensation level to the level of free convection. This seems reasonable since only short segments of the curve are used.

The interpolation for (*PX*) is found from

$$TLCL=\left(\frac{TLCL-TLFC}{PLCL-PLFC}\right)(PLCL-PX)$$

$$=TM-\frac{0.288TM}{PM}(PM-PX).$$

Solving for $(P\chi)$, we obtain

$$P\chi=\frac{TM-TLCL+PLCL\left(\dfrac{TLCL-TLFC}{PLCL-PLCF}\right)-0.288PM}{\left(\dfrac{TLCL-TLFC}{PLCL-PLFC}\right)-\dfrac{0.288TM}{PM}}. \tag{13}$$

REFERENCES

Berry, F. A., Jr., E. Bollay and Norman R. Beers, Eds., 1945: *Handbook of Meteorology*. New York, McGraw-Hill Book Co., p. 343.

Foster, Donald S., 1958: Thunderstorm gusts compared with computed downdraft speeds. *Mon. Wea. Rev.*, **86**, 91–94.

——, and Ferdinand C. Bates, 1956: A hail size forecasting technique. *Bull. Amer. Meteor. Soc.*, **37**, 135–141.

U. S. Weather Bureau, 1956: Forecasting tornadoes and severe thunderstorms. Forecasting Guide No. 1, Washington, D. C.